Implementation of AW Compensation Based-on Multifiltering for Fault Diagnosis

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Abstract—An approach for implementation of a control system with anti-windup compensation using fault detection multifiltering is shown. The residual signal used for anti-windup compensation is obtained from a filter bank (i.e. multifiltering) for fault detection, thus its explicit measurement is unnecessary. This residual signal is considered as a fault, in order to design the fault detection filter. Filter bank synthesis is achieved by means of LMI based robust control techniques. Additional LMI restrictions have been used in the optimization process, to guarantee better performance. This method allows to consider multiobjective performance indexes, for different actuator saturation conditions, in the same fashion as in fault diagnosis problems. A numerical example to verify the proposed implementation method effectiveness is shown.

I. INTRODUCTION
The necessity of safe and efficient control systems has generated a wide research to guarantee such conditions. One of the most common challenges in the literature has been to ensure stability and performance conditions when there are damages or any limitation in the functionality of the control systems. The formal condition that considers the stability is referred to Fault Tolerant Control Systems (FTCS) [1], [2], [3], and the performance condition is related to Bounded Control techniques. For bounded control the Anti-Windup strategies (AW) can be applied, which try to compensate the negative effects produced by saturation, or by the controller switching in different operation points for some controllers.

As consequence of the actuator physical limitations and changes in the operation conditions, given for production rules, it is common to find in the practice, an inconsistence between the process control input and the controller output. The inconsistence between these signals has as result that the controller states are forced to their upgrade, which produces undesired effects like large overshoot, adding even more actuators saturation, in some extreme cases producing an unstable closed loop [4]. This effect is called windup, and its solution has been studied (in an empirical form initially) since many decades ago [5].

The windup problem can be handled by means of compensation, where in a first stage, the control system is designed without taking into account the restrictions; and in a second stage, some compensation scheme is added, with the purpose of minimizing the effect of limitations and commutations. The last outlined focus has been denominated the anti-windup bumpless transfer problem (AWBT), [6].

In general, the the AWBT problem solution, using a compensation mechanism, requires a residual signal obtained by the difference between the controller output and the actuator nonlinear output. Measuring of such a residual constitutes an additional problem, from the point of view of the installation of the selected compensation scheme, which in many cases is difficult to achieve and demands the use of actuator models, by example. The use of models has the additional drawback of not generating the appropriate residuals for changes in the operation of actuators, this is, the model will not reflect any change in the saturation level. Some approaches have been proposed in order to implement AW compensation in industrial processes, [7], [8], [9].

On the other hand, FTC systems are formed by Fault Detection and Isolation (FDI) elements, which determine when and where a fault is done, taking as a basis the information contained in the residuals [10], [11]. The way as these residuals are generated, varies according to the application [12]. Generally, if detectability and separability conditions are fulfilled, we design a single FDI filter for diagnosis, whereby some restrictive and conservative results may be obtained [13]. Another approach is to use multiple filtering, in this case we build a filter bank which produces residuals for any particular failure [14].

In this contribution we solve the problem using a detection filter bank (multifiltering), oriented to the practical implementation of AW compensation techniques, where each filter is designed satisfying performance indexes characterized as LMIs.

As is known, a main component of FTC systems is the decision mechanism, which takes the information contained in the residuals and decides if there is failure presence or not. FTC systems are classified as passive, which use some techniques as robust control in a single closed loop design, making the control system resistant to a relative small fault group (in some cases taken as perturbations); on the other hand there exist active FTC systems, which can make changes on some loop elements. Generally the controller is changed in different ways, either in the form of new controller parameters or a different controller structure. The FTC philosophy is used in order to implant AW compensation techniques.

As has been mentioned, the AW approaches are mainly treated in a two step methodology: 1) To design a
controller assuming normal operating conditions, 2) To design some compensation (dynamic or static) which keeps the operation conditions under some situation, as windup problems for instance, or overshoots because of controller operation states changes [4]. In this work we will use the latter approach. The research and information about control problems under actuators restrictions is wide [15]. The approach used for AW compensation gain synthesis is given in [16].

In all cases, the signal used to implement the AW compensation must be measured directly from the process, generally as the difference between the controller output signal (which is almost always known beforehand) and the actual plant input. This situation is depicted in the Fig. 1.

![Fig. 1. Anti-windup compensated system.](image1)

In some cases, measurement of needed signal for compensation is not possible, nor is the addition of process instrumentation economically feasible.

The use of actuator models is presented in [17], which reproduce the actuators performance. The main problem with this approach as in any model based one, is that a limited model capacity may be achieved. Changes in actuator operating conditions, due to aging or deterioration, reduce the model precision. Additionally, any model always presents some uncertainty level.

The proposed solution consists in obtaining this signal by means of estimation using FDI filters, in such a way that the residual signal produced by them is used to implement the AW compensation [18]. Fig. 2 presents the proposed implementation methodology.

![Fig. 2. Anti-windup compensated system using FDI.](image2)

The main advantage of filter bank use is, that the conservative results present in a single filter design, are reduced in part, given that for every filter designed, i.e., for every failure or actuator, it is possible to use different conditions. Moreover, it allows us to consider many performance objectives according to saturation conditions.

This document is organized as follows. Section 2 resumes briefly an approach for AW compensation design, which is based on LMI, and has been presented in [16]. In Section 3, the SDI (Saturation Detection and Isolation) proposal is presented, as well as the main result for its synthesis. Finally, Section 4 shows a numerical example as an application to the proposed AW implementation alternative. Used notation is standard, in any different case it will be noted there.

II. AW COMPENSATION DESIGN

In this section, a short review of the AW compensation design technique is presented.

Let us assume we have the system

\[
\dot{x}(t) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} u(t) \\ 0 \end{bmatrix},
\]

and a dynamic output feedback controller

\[
\begin{align*}
\dot{x}_k(t) &= A_x x_k(t) + B_k y(t) \\
u_k(t) &= C_k x_k(t) + D_k y(t)
\end{align*}
\]

is done. Let us assume now the control signal is under saturation, i.e.,

\[
-u_{0(i)} \leq u(i) \leq u_{0(i)}, \quad u_{0(i)} > 0, \quad i = 1, \ldots, m,
\]

now, the control signal reads

\[
u(t) = \text{sat}(u_k(t)) = \text{sat}(C_k x_k(t) + D_k y(t)).
\]

To reduce the negative effects produced by saturation, we introduce a compensation (see Figure 1)

\[
c_k(t) = E_k [\text{sat}(u_k(t)) - u_k(t)],
\]

then, the closed loop system is

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B \text{sat}(u) \\
y(t) &= C x(t) \\
\dot{x}_k(t) &= A_x x_k(t) + B_k y(t) + E_k [\text{sat}(u_k(t)) - u_k(t)] \\
u_k(t) &= C_k x_k(t) + D_k y(t),
\end{align*}
\]

which, defining an extended state vector \( \xi(t) = [x(t)^T \quad x_k(t)^T]^T \in \mathbb{R}^{n + n_k} \) and matrices

\[
A = \begin{bmatrix} A + BD_k C & BC_k \\ B_k C & A_k \end{bmatrix}, \quad B = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad R = \begin{bmatrix} 0 \\ I_{n_k} \end{bmatrix}, \quad K = [D_k C \quad C_k],
\]

can be expressed as

\[
\dot{\xi}(t) = A \xi(t) - (B - RE_k) \psi(K \xi(t)).
\]

With all this considerations in mind, the next result is given.

Proposition 2.1: If there exist a symmetric positive definite matrix \( W \in \mathbb{R}^{Q(n + n_k)} \), and a matrix \( Y \in \mathbb{R}^{Q(n + n_k)} \),
a particular case where the faults detected are saturations. Defining an estimation error as
\[ e(t) = x(t) - \hat{x}(t), \] (13) thus, the estimator dynamic is given by
\[ \dot{\hat{e}}(t) = \dot{x}(t) - \hat{x}(t) \]
\[ = (A - D C_2) e(t) + B_1 w(t) + B_2 \psi(u(t)). \] (14)
Additionally, the output prediction error reads
\[ e_z(t) = C_1 x - C_1 \hat{x}(t) = C_1 e(t). \] (15)
From (14), the requirements that must be satisfied in the estimator gain \( D \) design are evident. In the procedure of state estimation, we wish that the estimated states be the same as the real ones, this is, the estimation error must be null in a finite time, i.e., taking \( \psi(t) = 0 \):
1) \( (A - D C_2) \) must be asymptotically stable.
2) The effect due to \( w(t) \) over \( e_z(t) \) perturbations must be minimized, assigning some norm \( (2, \infty) \).
In this case the estimation error never will be null, because always there will be some perturbations influence. This fact implies the bound existence, within which we consider that there is a situation normal or may exist a fault presence.
Provided that, in an \( i \)-input system, we have the same actuators number, the separation problem, in this case, consists of determining which actuator is saturated. It is worth noting that the saturation problem may appear in many actuators at the same time, and even so the residual signal must be separated.

A. The separation problem: Multifiltering

The main problem now, is that all the information of a fault presence (saturation in this case) is contained in the error signal, but in this instance it is not possible to identify in which actuator the problem is. The solution presented here, is to design different gains associated to different faults, obtaining in this way a filter bank for residual generation.
Using this framework, the filter bank is given by
\[ \hat{z}_i(t) = A \hat{x}_i(t) + B_2 u(t) + D_i (y(t) - C_2 \hat{x}(t)) \]
\[ \hat{z}_i(t) = C_1 \hat{x}_i(t), \quad i = 1, \ldots, m, \] (16)
where every \( \hat{z}_i(t) \) constitutes an estimated controlled output, each one obtained by means of \( D_i \) gain. Every filter is designed in such a way that attenuates external perturbations and detects only one fault in a particular actuator.
Now, the prediction errors for the filter bank are
\[ \hat{e}_i(t) = A e_i(t) + B_2 \hat{w}(t) + B_2 \psi_i(t), \]
\[ e_{zi}(t) = C_1 e_i, \quad i = 1, \ldots, m, \] (17)
with
\[ B_2 = \begin{bmatrix} B_{11} & B_{20} \end{bmatrix}, \quad \hat{w}(t) = \begin{bmatrix} w(t) \\ \psi_0(t) \end{bmatrix}. \] (18)

\( B_{1i} \) and \( \psi_i(t) \) are the fault signature and mode respectively, for the \( i \)-th designed filter. We note that the \( i \)-th
fault is associated to the \(i\)-th actuator, that is the reason to have the \(i\)-th column of \(B_2\) matrix, as fault signature. \(B_{20}\) and \(\psi_0(t)\) contain the rest of fault signatures and modes respectively.

It is worth noting that the additional modes and signatures, different to the filter associated, i.e., the currently designed one, are inside the extended perturbation matrix \(B_i\) and the extended exogenous perturbation signals \(\tilde{w}(t)\in \mathbb{R}^{n+f-1}\) respectively. It means, those faults are taken as perturbations in the design process, and consequently their effect will be minimized on the same way.

In this way, the generated residual by this filter, only corresponds to the fault \(\psi_i(t)\), since the effect produced by the another faults is attenuated by \(D_i\) matrix as a result of the optimization process.

Particularly it is desired

\[
\|G_{\tilde{w} \rightarrow \epsilon_1}(s)\| = C_1(sI - A)^{-1}B_1 \leq \gamma \tag{19}
\]

\[
\|G_{\psi \rightarrow \epsilon_1}(s)\| = C_1(sI - A)^{-1}B_{2i} \leq \gamma \tag{20}
\]

The reason for the second inequality is to maintain the system \(G_{\psi \rightarrow \epsilon_1}(s)\) gain (in this case expressed as a \(\infty\)-norm) as high as it is possible, in such a way that the fault will not be confused with effects produced by perturbations.

To solve the filter design problem, we use a classic result known as Bounded Real Lemma and LMI stability of the roots, resumed on the next result.

**Proposition 3.1:** There exist a detection filter for the \(i\)-th fault such that \((A - D_iC_i)\) is asymptotically stable, \(\|G_{\tilde{w} \rightarrow \epsilon_1}(s)\| < \gamma\) with eigenvalues placed on the left of \(\alpha_i\), if only if, there exist matrices \(P_i = P_i^T > 0\in \mathbb{R}^n\) \(W_i \in \mathbb{R}^{n \times s}\) \(\psi_i(t)\), such that

\[
\begin{bmatrix}
ATP_i + P_iA - WC_2 - C_2^TW_i^T & P_iB_i & C_i^T \\
B_i^TP_i & - \gamma I & 0 \\
C_i & 0 & - \gamma I
\end{bmatrix} < 0.
\]

\[
ATP_i + P_iA - WC_2 - C_2^TW_i^T + 2\alpha_iP_i < 0 \tag{21a}
\]

\[
\alpha_i > 0
\]

is satisfied. In that case, the estimator gain is given by \(W_i = P_iD_i\).

**Proof**

Let us assume (21) is feasible and there is a solution to the LMI system, making the variable change \(W_i = P_iD_i\) we have

\[
\begin{bmatrix}
ATP_i - C_2D_i^TP_i + P_iA - P_iD_iC_2 & P_iB & C_i^T \\
B_i^TP_i & - \gamma I & 0 \\
C_i & 0 & - \gamma I
\end{bmatrix} < 0
\]

\[
ATP_i - C_2D_i^TP_i + P_iA - P_iD_iC_2 + 2\alpha_iP_i < 0
\]

which is equivalent to

\[
\begin{bmatrix}
(A - D_iC_i)^TP_i + P_i(A - D_iC_i) & P_iB & C_i^T \\
B_i^TP_i & - \gamma I & 0 \\
C_i & 0 & - \gamma I
\end{bmatrix} < 0
\]

\[
(A - D_iC_i)^TP_i + P_i(A - D_iC_i) + 2\alpha_iP_i < 0
\]

From the bounded real lemma, we have \((A - D_iC_i)\) is stable on the LMI region \(\mathcal{R}(\Re(s) < -\alpha_i, \alpha_i > 0)\) and

\[
\|G_{\tilde{w} \rightarrow \epsilon_1}(s)\|_{\infty} < \gamma, \text{ if only if, the last inequalities are satisfied [20], [21].}
\]

**Remark 3.1:** Because of the performance index used, only takes in consideration the transfer function \(G_{\tilde{w} \rightarrow \epsilon_1}(s)\) for the optimization process, there is not any warranty of (20) satisfaction, i.e., minimizing

\[
\|G_{\tilde{w} \rightarrow \epsilon_1}(s)\|_{\infty}, \|G_{\psi \rightarrow \epsilon_1}(s)\|_{\infty} \quad \text{may be seen in an undesired way affected too. To prevent this, it must be sufficed that} \quad B_i, \quad B_{2i}, \quad \text{map different subspaces, i.e.,}
\]

\[
OB_i \cap OB_{2i} = \emptyset.
\]

Presented the necessary theoretical results, next a numerical example is shown.

**IV. NUMERICAL EXAMPLE**

Let us consider the LTI system given by [22]:

\[
x(t) = \begin{bmatrix}
0 & 0 & 1.132 & 0 & 0 & 0.0705 \\
0 & 0 & 0 & 0 & 0 & 0.0539 \\
0 & 0 & 0.0485 & 0 & -0.5568 & 0 \end{bmatrix} x(t) + \begin{bmatrix}
1 \\
0 \\
0 \\
-1 \\
0 \\
0
\end{bmatrix} u(t) + \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} w(t) + \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} u(t).
\]

we made a slight change to introduce a perturbation over the states. We wish to design a control system with AW compensation.

The fist step is the controller design, to this aim we use LMI based robust control techniques [23], [24]. In the design we want \(\|G_{\tilde{w} \rightarrow z}(s)\| < \gamma = 2\), getting as a solution:

\[
A_k = \begin{bmatrix}
-1.0369 & -0.6843 & 1.5554 & -0.3236 \\
0.011161 & -2.4448 & 2.8355 & 8.6661 \\
-0.059225 & 1.3007 & -0.9408 & 3.6631 \\
-0.134611 & -2.6047 & 0.1608 & -17.654 \\
0.516831 & -1.5789 & 0.96026 & -26.37 \\
-0.064630 & -0.12374 & 0.23996 & -2.5902 \\
-0.050615 & -0.22148 & -0.7903 & -0.23992 \\
-0.178923 & -0.07174 & 0.43488 & 0.08874 \\
\end{bmatrix}
\]

\[
b_k = \begin{bmatrix}
10.0254 & -1.2423 & -0.03880 \\
-1.6054 & -1.425 & 2.3411 \\
7.0092 & -0.54082 & 10.12 \\
8.1382 & -2.4019 & -3.8993 \\
-8.2445 & -1.1757 & -11.866 \\
9.5968 & 1.1757 & -10.89 \\
-0.050855 & 14.564 & 1.2069 \\
-1.0328 & -0.97888 & -9.7126 \\
\end{bmatrix}
\]

\[
c_k = \begin{bmatrix}
0.0002197 & -0.085143 & 0.087503 & -1.9036 \\
0.0062082 & -0.14871 & 0.03342 & -0.02668 \\
-0.057531 & -0.14771 & 0.24015 & -5.0559 \\
0.52531 & 16.599 & -1.0394 & -34.339 \\
0.28422 & 2.6065 & -0.1344 & -4.9336 \\
-0.37893 & 21.217 & -2.3105 & -47.861
\end{bmatrix}
\]

For the closed loop system, \(\|G_{\tilde{w} \rightarrow z}(s)\| = 1.5290\).

Now, let

\[
\Xi_0 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
15 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 \\
15 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 \\
15 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 \\
15 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 \\
15 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05
\end{bmatrix}
\]

be the basis set over which the ellipsoid will be maximized , and \(U_0 = \begin{bmatrix} 15 & 100 & 35 \end{bmatrix}\) the actuator bounds.
According to proposition 2.1, solving the LMI set, the AW compensation gain is:

\[
E_c = \begin{bmatrix}
-71.569 & -0.55025 & 9.2066 \\
-3.1085 & 21.838 & 5.2673 \\
80.074 & 8.4381 & 33.759 \\
-110.81 & -2.1256 & -33.187 \\
-87.937 & -0.13526 & -11.873 \\
-88.407 & 1.2082 & -7.8538 \\
129.32 & 38.671 & 30.198 \\
-59.224 & -1.956 & -6.5699
\end{bmatrix}
\]

For the SDI filters design purpose, the matrices are distributed on the next way. In every design case, the extended perturbation and signature matrices are given by

\[
B_1 = \begin{bmatrix}
0.001 & 0 \\
0 & 0 \\
0 & 0 \\
0.001 & -1.6650 \\
0 & 0.0732 \\
0.001 & 0 \\
0 & 0 & 0 \\
0 & 0 & -1.2000 \\
0 & 0 & 0 \\
-0.001 & 4.4190 \\
0 & 0 & 1.5750
\end{bmatrix},
B_2 = \begin{bmatrix}
0 \\
-1.1200 \\
0 \\
0 & 4.4190 \\
0 & 1.5750 \\
0 & 0 & 0 \\
0 & 0 & -1.6650 \\
0 & 0 & 0 \\
0 & 0 & -0.0732
\end{bmatrix}
\]

In our design, only actuators 1 and 3 are taken into account, i.e., only these actuators are limited, and then they will be compensated. According to proposition 3.1, taking \( \gamma_1 = \gamma_2 = 1 \times 10^{-3} \), and \( \alpha_{1,2} \approx 230 \), the solutions, i.e., the detection filter gains are given by

\[
D_1 = \begin{bmatrix}
0.0002 & -0.0000 & -0.0000 \\
-0.0000 & 0.0001 & -0.0000 \\
-0.0000 & 0.0000 & 0.0012 \\
-0.0230 & 0.0417 & 4.6486 \\
-0.0459 & 0.0505 & 0.1956
\end{bmatrix} \times 10^7
\]

\[
D_2 = \begin{bmatrix}
0.0001 & -0.0000 & 0.0001 \\
0.0006 & 0.0089 & -0.0010 \\
-0.0007 & 0.0000 & 0.0015 \\
-0.3166 & -0.3140 & 0.6932 \\
-0.6135 & -0.0884 & 1.2369
\end{bmatrix} \times 10^9
\]

For evaluation of performance, some simulations have been released. Fig. 3(a), shows a closed loop simulation without actuator limits. As can be seen, we are using different references for the output, with the purpose of generate strong changes in the control signal. Fig. 3(c) and 3(b) show the behavior for the saturated actuators with and without compensation respectively.

Such as can be seen in the Fig. 3(b), for the saturated-without-compensation case the system is unstable, whereas for the without-saturation and saturated-with-compensation case, Fig. 3(a) and 3(c), the performance is similar. Due to the reference changes, the actuators saturation is present, see Fig. 4(a) (lowest part). The implementation of the AW compensation has been possible due to the adequate design of the SDI filters, which have generated the estimated residual signal shown in Fig. 4(b), which is similar to measured residual signal (upper part).

V. CONCLUDING REMARKS

In this contribution, an approach for practical implementation of AW compensation has been presented. The method consists of estimating the residual signal between the control output and the actuator nonlinear output, applying fault detection and isolation techniques. In this case, the residual signal was considered as one fault. Detectability and separability conditions were considered in order to distinguish which actuators were saturated. A FDI filter bank scheme (multifiltering), was applied to obtain the residual signals when some actuators were saturated. The use of a FDI filter bank allowed to handle the variations in the performance of the actuators. This AW compensation implementation method, is robust with respect to changes in the saturation limits of the actuators. The filters were designed using performance indexes defined as LMI restrictions. Moreover, in order to improve the performance, additional conditions have
example, the proposed method effectiveness has been applied. An interesting aspect represents the fact that multiobjective techniques can be used in this framework, according to different saturating conditions or to the different frequencies that may be contained in the residual and disturbance signals. By means of a numerical example, the proposed method effectiveness has been validated.

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REFERENCES