

Smith-Predictor Compensator for a Delayed Omnidirectional Mobile Robot

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Abstract—This paper deals with the discrete time control of an omnidirectional mobile robot subject to transport delays. A discrete-time model of the vehicle is obtained by considering an exact discretization of the continuous time model of the robot where the time delay induced by a communication network is considered. The path-tracking problem is addressed by means of a prediction strategy based on the well known Smith predictor compensator. The nonlinear nature of the omnidirectional mobile robot induces an approximate estimate of the future values of the systems that are used together with a feedback linearization approach to solve the tracking problem.

I. INTRODUCTION

Transport delay problems in mobile robots have not been yet extensively discussed. In particular, the path-tracking problem of a mobile robot including the effects of a communication network delay has not been extensively treated. A remotely controlled mobile robot is an example of a system with time delay where the delay effect on the stability of the overall system can not be neglected.

The necessity to consider the transport delay problem has motivated the use of discrete-time models that allows the analysis of the effects of the communication network delay and the synthesis of discrete-time controllers designed to compensate such effects. The complexity of the problem increases especially when the transport delay is included in the model due to the nonlinear nature of mobile robots.

There are several works dealing with discrete-time models of a mobile robot, especially for systems of the type (2,0). In [1] the authors propose a discrete-time sliding mode control based on an approximate discrete-time model of the robot. In [2] the exact discrete-time model of a mobile robot (2,0) is obtained by direct integration and a discrete-time nonlinear control scheme is developed. In [3] the authors present the transforming of the original kinematics system in chained form through an appropriate feedback. The obtained equations are closed-form integrable, thereby yielding a linear discrete-time model which provides a suitable odometric prediction. In [4], [5] the authors consider an approximate discrete-time model of a mobile robot that is controlled remotely introducing the effects of communication network delay. Specifically, in [4] a control law that stabilizes the mobile

robot in the presence of a time delay less than the sampling period of the system is obtained.

The problem that arises when trying to control a mobile robot remotely resides in the fact that the communication network often induces transport delays. If an amount of time passes between the generation of the control signal in the controller and its effective application in the remote system, poor performance may be obtained [6], or in some cases, the system may even become unstable [4], [5], [7].

Some considerations regarding the transport delay in the communication network when the time delay τ is less than the sampling period T are presented in [8], [9]. The practical implementation of a control law for most mobile robots implies the necessity to consider the time delay due to the communication network which links the sensors and actuators in the robot and the controller placed in a remote location. A schematic representation of this configuration is shown in Figure 1.

The path-tracking problem of an omnidirectional mobile robot subject to transport delays is addressed in this work. An exact discrete-time delayed model of the original continuous time delayed system is obtained when the induced time delay is larger than the sampling period of the system.

The path-tracking problem is solved by means of a discrete-time nonlinear control scheme based on the exact discrete-time model (including the time delay caused by the propagation of the control signals) and by considering a feedback linearization approach which produce in a first attempt a noncausal solution. The estimation of the future values needed by the linearization strategy is obtained in an approximated way by considering a nonlinear generalization of the well known (for the linear case) Smith predictor compensator [10]. In this way, assuming the availability at time t of the state at time $t + \tau$ is possible to implement the proposed strategy in a causal manner. The present proposal is based on the original works [11], [12] in the field of chemical processes.

This paper is organized as follows: In Section II the kinematics model of an omnidirectional mobile robot and its equivalent discrete time representation are presented. In Section III the path tracking problem is analyzed providing a discrete-time solution by means of a feedback linearization approach. The Smith predictor compensator

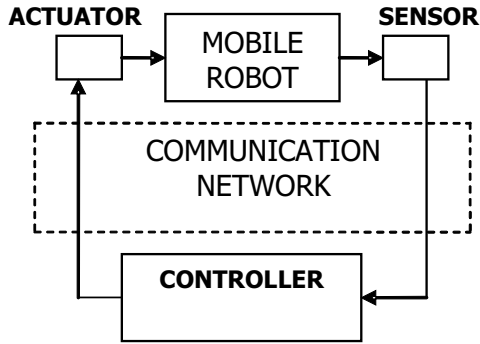


Fig. 1. Communication network

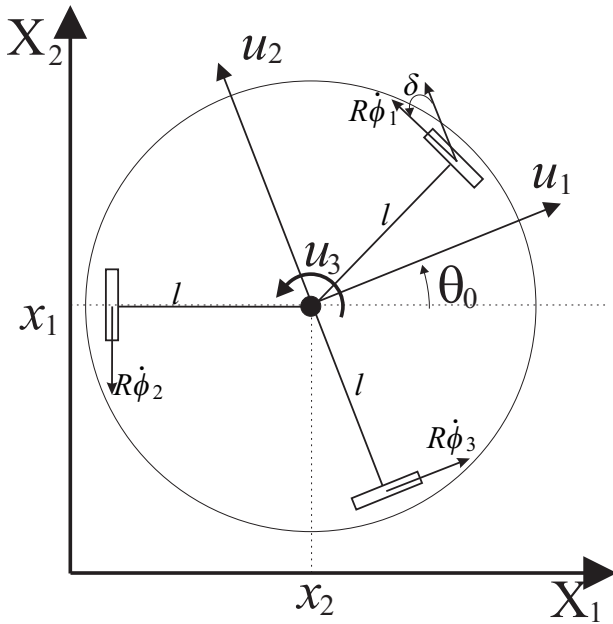


Fig. 2. Omnidirectional mobile robot

strategy is presented in Section IV. Some simulation results are presented in Section V and finally, in Section VI some conclusions are given.

II. OMNIDIRECTIONAL MOBILE ROBOT

The kinematics model of an omnidirectional mobile robot can be easily obtained by considering the geometric representation given in Figure 2. The velocity components with respect to the axis $X_1 - X_2$ are obtained as [13], [14],

$$\begin{aligned} \dot{x}_1 &= u_1 \cos \theta_0 - u_2 \sin \theta_0 \\ \dot{x}_2 &= u_1 \sin \theta_0 + u_2 \cos \theta_0 \\ \dot{\theta}_0 &= u_3 \end{aligned} \quad (1)$$

where the point (x_1, x_2) is the position of the center of the robot on the axes $X_1 - X_2$ and θ_0 is the angular position with respect to the axis X_1 . The input signals are given by u_1, u_2 and u_3 , with u_1, u_2 two orthogonal velocity vectors where u_1 is aligned with the reference axis of the robot, u_3 correspond to the rotational velocity of the robot.

From Figure 2 it is also easy to see that the wheel velocities are related to the components over the axis $X_1 - X_2$ and the rotational velocity as [15],

$$\begin{bmatrix} R\dot{\phi}_1 \\ R\dot{\phi}_2 \\ R\dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} -\sin(\theta_0 + \delta) & \cos(\theta_0 + \delta) & l \\ -\sin(\theta_0 - \delta) & -\cos(\theta_0 - \delta) & l \\ \cos \theta_0 & \sin \theta_0 & l \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta}_0 \end{bmatrix}$$

where ϕ_i is the rotational velocity of each wheel and R is the radius, l gives the distance between each wheel and the center of the vehicle and δ is the orientation of the wheel with respect to axes of the vehicle.

Notice also that for a possible implementation, the relation that exists between the input signal of the system u_1, u_2 and u_3 and the angular velocity of each wheel is given by:

$$\begin{bmatrix} R\dot{\phi}_1 \\ R\dot{\phi}_2 \\ R\dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} -\sin \delta & \cos \delta & l \\ -\sin \delta & -\cos \delta & l \\ 1 & 0 & l \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}.$$

The time delay induced by the communication network can be modeled by a constant delay at the input signal in the way,

$$\begin{aligned} \dot{x}_1 &= u_1(t - \tau) \cos \theta_0 - u_2(t - \tau) \sin \theta_0 \\ \dot{x}_2 &= u_1(t - \tau) \sin \theta_0 + u_2(t - \tau) \cos \theta_0 \\ \dot{\theta}_0 &= u_3(t - \tau). \end{aligned} \quad (2)$$

A. Discrete-time model

The exact discrete time model of the delayed robot can be easily obtained by the direct integration of the equations given in (2). For doing this, consider a positive constant $T \neq 0$ as a sampling period and define t_k as the time interval between two sampling instant as

$$t_k = t \in [kT, kT + T)$$

where: $k = 0, 1, 2, 3, \dots$

Assume also that the input signal $u(t)$ is constant between sampling instants, i.e., $u(t) = u(kT)$. Under these conditions, the direct integration of the last equation of (2) produces,

$$\theta_0(t) = \theta_0(kT) + [t - kT]u_3(kT - \tau). \quad (3)$$

Substituting this equation into the first and second equations of (2) it is possible to obtaining,

$$\begin{aligned} x_1(t) &= x_1(kT) \\ &+ \frac{u_1(kT - \tau)}{u_3(kT - \tau)} (\sin(\theta_0 + Tu_3(kT - \tau)) - \sin \theta_0) \\ &+ \frac{u_2(kT - \tau)}{u_3(kT - \tau)} (\cos(\theta_0 + Tu_3(kT - \tau)) - \cos \theta_0) \end{aligned} \quad (4)$$

and

$$\begin{aligned} x_2(t) &= x_2(kT) \\ &- \frac{u_1(kT - \tau)}{u_3(kT - \tau)} (\cos(\theta_0 + Tu_3(kT - \tau)) - \cos \theta_0) \\ &+ \frac{u_2(kT - \tau)}{u_3(kT - \tau)} (\sin(\theta_0 + Tu_3(kT - \tau)) - \sin \theta_0) \end{aligned} \quad (5)$$

In order to simplify presentation of the subsequent developments, where no confusion arise, it will be adopted the following notation,

$$\zeta = \zeta(kT), \zeta^\pm = \zeta(kT \pm T), \zeta^{[\pm n]} = \zeta(kT \pm nT). \quad (6)$$

After some manipulations of the discrete time model given by equations (3)-(4)-(5), the exact discrete time model of the omnidirectional mobile robot can be expressed by using the notation given in (6) as,

$$\begin{aligned} x_1^+ &= x_1 + 2u_1(kT - \tau)\gamma \cos\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ &\quad - 2u_2(kT - \tau)\gamma \sin\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ x_2^+ &= x_2 + 2u_1(kT - \tau)\gamma \sin\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ &\quad + 2u_2(kT - \tau)\gamma \cos\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ \theta_0^+ &= \theta_0 + u_3(kT - \tau) \end{aligned} \quad (7)$$

where the function $\gamma(u_3(kT - \tau))$ satisfies,

$$\gamma(u_3(kT - \tau)) = \begin{cases} \frac{\sin(\frac{T}{2}u_3(kT-\tau))}{u_3(kT-\tau)} & \text{if } u_3(kT - \tau) \neq 0 \\ \frac{T}{2} & \text{if } u_3(kT - \tau) = 0. \end{cases} \quad (8)$$

III. PATH-TRACKING PROBLEM

In this section is presented a non causal solution (anticipative) to the path-tracking problem for a mobile robot of the type (3,0). The solution will be based on a feedback linearization approach. Consider the exact discrete time model presented in equation (7) and the output given by the signals,

$$y = h(x) = \begin{bmatrix} h_1(x) \\ h_2(x) \\ h_3(x) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \theta_0 \end{bmatrix}. \quad (9)$$

Following a standard procedure [16], [17], the forward-shift of the output (9) will produce,

$$\begin{aligned} h_1^+ &= x_1 \\ &\quad + 2u_1(kT - \tau)\gamma(u_3(kT - \tau)) \cos\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ &\quad - 2u_2(kT - \tau)\gamma(u_3(kT - \tau)) \sin\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ h_2^+ &= x_2 \\ &\quad + 2u_1(kT - \tau)\gamma(u_3(kT - \tau)) \sin\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ &\quad + 2u_2(kT - \tau)\gamma(u_3(kT - \tau)) \cos\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ h_3^+ &= \theta_0 + Tu_3(kT - \tau). \end{aligned} \quad (10)$$

Considering now a new input signal $v = [v_1 \ v_2 \ v_3]^T$ such that the closed-loop system satisfies $h^+(x, u(kT - \tau)) = v$, this is,

$$\begin{bmatrix} h_1^+(x, u(kT - \tau)) \\ h_2^+(x, u(kT - \tau)) \\ h_3^+(x, u(kT - \tau)) \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (11)$$

it is possible to obtain from equation (10),

$$\begin{aligned} u_1(kT - \tau) &= \frac{1}{2\gamma(u_3(kT-\tau))} \times \\ &\quad \times \{ [v_1 - x_1] \cos\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ &\quad + [v_2 - x_2] \sin\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \} \\ u_2(kT - \tau) &= \frac{1}{2\gamma(u_3(kT-\tau))} \times \\ &\quad \times \{ [v_2 - x_2] \cos\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \\ &\quad - [v_1 - x_1] \sin\left(\theta_0 + \frac{Tu_3(kT-\tau)}{2}\right) \} \\ u_3(kT - \tau) &= \frac{v_3 - \theta_0}{T}. \end{aligned} \quad (12)$$

Remark 1: Notice that feedback (12) it is well defined since the function $\gamma(u_3(kT - \tau))$ is different from zero for all x as can be seen from equation (8).

In order to solve the path-tracking problem consider now the auxiliary inputs v_1 , v_2 and v_3 defined as,

$$\begin{aligned} v_1 &= x_{1d}^+ - k_1 e_1 \\ v_2 &= x_{2d}^+ - k_2 e_2 \\ v_3 &= \theta_{0d}^+ - k_3 e_3 \end{aligned} \quad (13)$$

with,

$$\begin{aligned} e_1 &= x_1 - x_{1d} \\ e_2 &= x_2 - x_{2d} \\ e_3 &= \theta_0 - \theta_{0d} \end{aligned} \quad (14)$$

where e_1 , e_2 and e_3 are the position errors of the states x_1 , x_2 and θ_0 respectively and the subscript d is used to denote the desired trajectories. k_1 , k_2 and k_3 are real constants such that the closed loop system (7)-(12) is stable.

Remark 2: It is very important to notice that equation (12) represent a noncausal feedback that can not be directly implemented on system (7). This fact represents the main drawback of the developments presented before that is a common issue in the case of input delay systems.

From Remark 2 it is clear that a practical feedback to be used on system (7) should be synthesized at time t or kT , in this case, feedback (12) it is described as,

$$\begin{aligned} u_1(kT) &= \frac{1}{2\gamma(u_3)} \{ [v_1(kT + \tau) - x_1(kT + \tau)] \times \\ &\quad \times \cos\left(\theta_0(kT + \tau) + \frac{Tu_3}{2}\right) \\ &\quad + [v_2(kT + \tau) - x_2(kT + \tau)] \times \\ &\quad \times \sin\left(\theta_0(kT + \tau) + \frac{Tu_3}{2}\right) \} \\ u_2(kT) &= \frac{1}{2\gamma(u_3)} \{ [v_2(kT + \tau) - x_2(kT + \tau)] \times \\ &\quad \times \cos\left(\theta_0(kT + \tau) + \frac{Tu_3}{2}\right) \\ &\quad - [v_1(kT + \tau) - x_1(kT + \tau)] \times \\ &\quad \times \sin\left(\theta_0(kT + \tau) + \frac{Tu_3}{2}\right) \} \\ u_3(kT) &= \frac{1}{T} \{ v_3(kT + \tau) - \theta_0(kT + \tau) \} \end{aligned} \quad (15)$$

where,

$$\begin{aligned} v_1(kT + \tau) &= x_{1d}(kT + \tau + T) - k_1 e_1(kT + \tau) \\ v_2(kT + \tau) &= x_{2d}(kT + \tau + T) - k_2 e_2(kT + \tau) \\ v_3(kT + \tau) &= \theta_{0d}(kT + \tau + 1) - k_3 e_3(kT + \tau) \end{aligned} \quad (16)$$

and,

$$\begin{aligned} e_1(kT + \tau) &= x_1(kT + \tau) - x_{1d}(kT + \tau) \\ e_2(kT + \tau) &= x_2(kT + \tau) - x_{2d}(kT + \tau) \\ e_3(kT + \tau) &= \theta_0(kT + \tau) - \theta_{0d}(kT + \tau) \end{aligned} \quad (17)$$

To overcome the non-causality problem of the feedback law (15) in what follows it will be considered a control strategy based on a Smith predictor compensator borrow from the linear case [10] and implemented initially in a nonlinear contest in [11], [12].

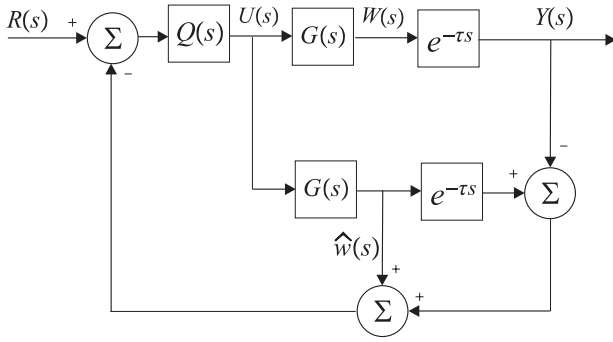


Fig. 3. Classical Smith Predictor scheme.

IV. SMITH PREDICTOR COMPENSATOR STRATEGY

A. Linear case

A classical Smith predictor compensator for a linear system is shown in Figure 3. The main idea behind this strategy is based on the modeling of the system as

$$W(s) = G(s)U(s) \quad (18a)$$

$$Y(s) = e^{-\tau s}W(s), \quad (18b)$$

and to design an estimator (predictor) for the intermediate signal $W(s)$ (not available for measurement). The smith predictor scheme use this signal in the controller $Q(s)$ depicted also in Figure 3, in order to compensate the effects of the time delay $e^{-\tau s}$ on the overall closed loop system. It is easy to see that the closed loop system for the compensation strategy of Figure 3 it is given by,

$$\frac{Y(s)}{R(s)} = \frac{G(s)Q(s)}{1 + G(s)Q(s)}e^{-\tau s}.$$

Under ideal conditions (*i.e.*, exact knowledge of the model), this prediction strategy allow to keep out of the closed loop the time delay term. Unfortunately, the classical Smith predictor scheme is able to deal only with stable plants [10]. Several modifications of the same strategy can only manage with some special class of unstable systems.

B. Nonlinear case

The non causality problem presented in the preceding section can be solved by obtaining the estimation of the future values of the state, there are different strategies for doing this, for instance, in many cases it is common practice to use the present value of the state instead of the future one, this fact produces a very simple approximation (zero order) that could degenerate the performance of the system. In the linear case, a very useful approach is the consideration of Smith predictor compensator [10], described above, that is capable of estimate the needed future values. In this work is considered a direct extension of this later strategy that consist on a dynamic compensator that approximately estimate the future values needed for the implementation of the non causal feedback law (15). In the continuous nonlinear case the first attempt to use a compensator of a Smith predictor type was done in [11] and later the same methodology was adapted for

the discrete time case in [12]. In what follows, this idea is implemented for the mobile robot considered in this work.

Consider the nonlinear discrete-time model subject to input delay given by,

$$\begin{aligned} x(k+1) &= f(x(k)) + g(x(k))u(k-\tau) \\ y(k) &= h(x(k)). \end{aligned} \quad (19)$$

The development of the Smith predictor compensator to be used with the feedback (15) is based on the following assumptions:

Assumption 3: System (19) is open loop stable.

Assumption 4: The free-delay system (19) ($\tau = 0$) has a stable zero dynamics.

Assumption 3 is common in the general development of the original Smith predictor for the linear case and can not be negligible in the nonlinear case [11]. Assumption 4 is a fundamental one in the case of a nonlinear controller based on a feedback linearization strategy [18] as the one considered in this work.

Note that in the case of the omnidirectional mobile robot (7) in closed loop with (15) the dimension of the resulting zero dynamics is zero.

The main purpose of the Smith predictor compensator is to provide the value of the state of the system at the instant $k+i$, this is, i units of time ahead. In this case the exact estate future value is given by,

$$x(k+i) = f(x(k+i-1)) + g(x(k+i-1))u(k+i-\tau-1). \quad (20)$$

Considering the developments of Henson and Seborg in [12], the approximation of the future state (20) can be computed in the case that $i = \tau$ by means of a dynamic compensator of the form,

$$\begin{aligned} \tilde{x}(kT+T) &= \tilde{f}(\tilde{x}(kT)) + \tilde{g}(\tilde{x}(kT))u(kT) \\ \hat{x}(kT+T) &= \tilde{f}(\hat{x}(kT)) + \tilde{g}(\hat{x}(kT))u(kT-\tilde{\tau}) \\ \delta x(kT) &= \tilde{x}(kT) - \hat{x}(kT) \end{aligned} \quad (21)$$

where the Smith predictor compensator-type provides the future approximate estimation $x^*(kT)$ as,

$$x^*(kT) = x(kT) + \delta x(kT)$$

A general control scheme based on the Smith predictor compensator for the nonlinear case is depicted in Figure 4.

The prediction strategy presented above has the following properties [12]:

- 1) It is possible to see that in the case of perfect modeling, this is, $\tilde{f} = f$, $\tilde{g} = g$, $\tilde{\tau} = \tau$ and $\hat{x}(t) = x(t)$, the prediction of the state satisfies $x^*(kT) = \tilde{x}(kT)$ that allows to consider that $x^*(k) = x(k+\tau)$ for all $k \geq 0$.
- 2) If the closed loop system is asymptotically stable, then $x^*(kT) \rightarrow x(kT+\tau)$ as $k \rightarrow \infty$.

Remark 5: It is clear that the proposed methodology can provide poor prediction results in the case of model mismatches or in the presence of disturbances.

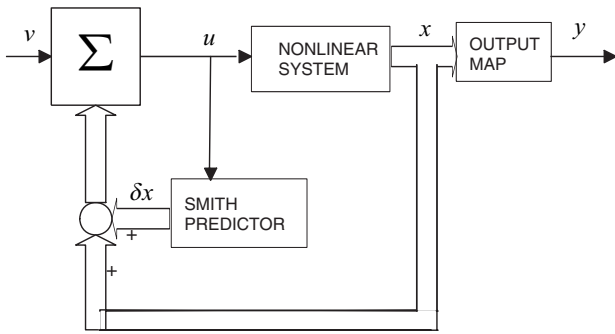


Fig. 4. Smith predictor structure

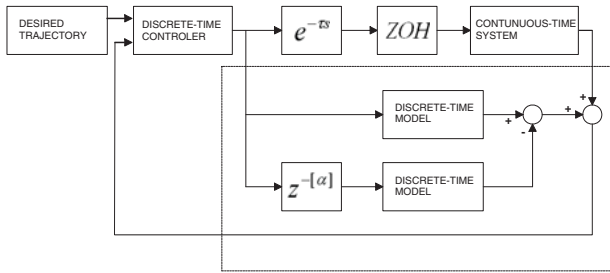


Fig. 5. Nonlinear Smith predictor compensator structure

V. SIMULATIONS RESULTS

In order to show the performance of the proposed methodology, in this section, several numerical simulations are carried out by considering the case of a straight line as a desired trajectory. In order to show a more realistic situation it is considered that the plant is the continuous time systems (1) while the smith predictor compensator is obtained from the exact discrete time system given by equation (7). The structure of the prediction control strategy adopted in this work is depicted in Figure 5.

For the discretization of the original model (1) and all the experiments it is considered a sampling period T of 100 msec.

A. Desire trajectory

The desire trajectory is obtained by means of $x_{2d}(t) = x_{1d}(t)$ with a time parameterization given by the polynomial

$$x_{1d}(t) = x_{1di} + (x_{1df} - x_{1di})\delta_t^5 [252 - 1050\delta_t] + 1800\delta_t^2 - 1575\delta_t^3 + 700\delta_t^4 - 126\delta_t^5$$

where $\delta_t = \frac{|t-t_i|}{t_f-t_i}$, with t_i, t_f are the initial and final time respectively and x_{1di}, x_{1df} been the initial and final position. Finally, the desired trajectory is generated by considering $x_{1di} = 0$ cm., $x_{2di} = 0$ cm. and $\theta_{0di} = \frac{\pi}{4}$ rad and the final time t_f of the experiment equal to 20 sec.

The initial conditions of the system are assumed to be $x_1(0) = 0, x_2(0) = 0$ and $\theta_0(0) = 0$. For the models (2) and (7) it is considered an input delay $\tau = 0.5$ sec. The parameters of the noncausal feedback (16) are given as

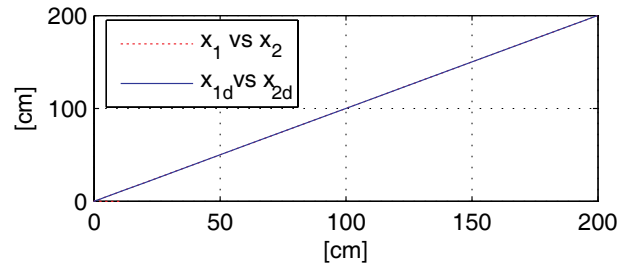


Fig. 6. Tracking of the desired and actual trajectory on the $X_1 - X_2$ plane

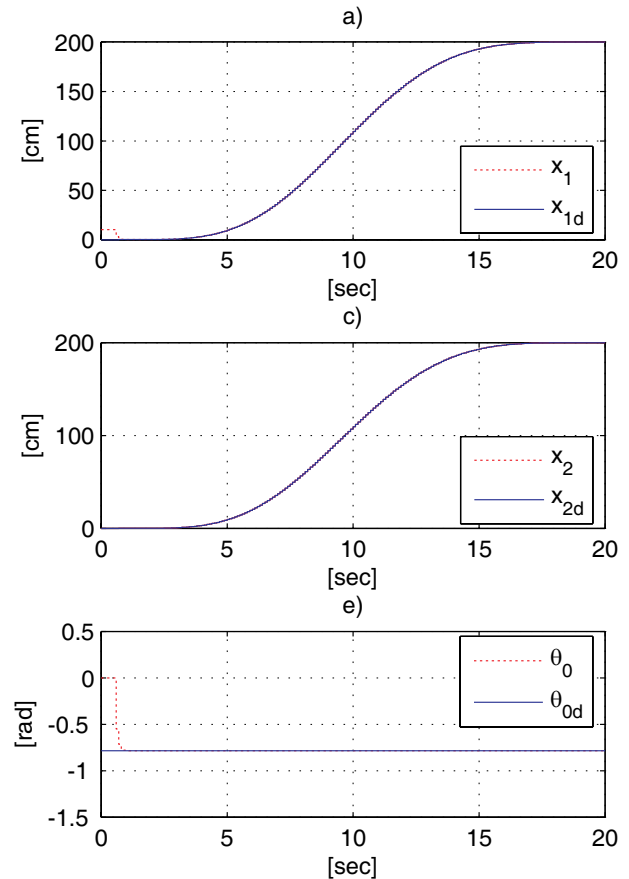


Fig. 7. State evolution versus desired values

$k_1 = 0.3, k_2 = 0.3$ and $k_3 = 0.3$. All the simulations are carried out on a Matlab-Simulink platform.

Figure 6 shows the evolution of the mobile robot on the plane $X_1 - X_2$ (dotted line) as well as the desire trajectory (solid line). The evolution of the states x_1 and its desired value x_{1d} are shown in Figure 7a, states x_2 and x_{2d} are shown in Figure 7b and the evolution of the angle θ_0 and θ_{0d} are depicted in Figure 7c. Finally, the control signal u_i and its delayed values $u_{ir} = u_i(t - \tau)$ for $i = 1, 2, 3$ are shown in Figure 8a, 8b, 8c. The numerical simulations confirm the effectiveness of the proposed strategy for the path-tracking control of an omnidirectional mobile robot.

VI. CONCLUSIONS

The discrete-time control of an omnidirectional mobile robot subject to transport delay is addressed in this work.

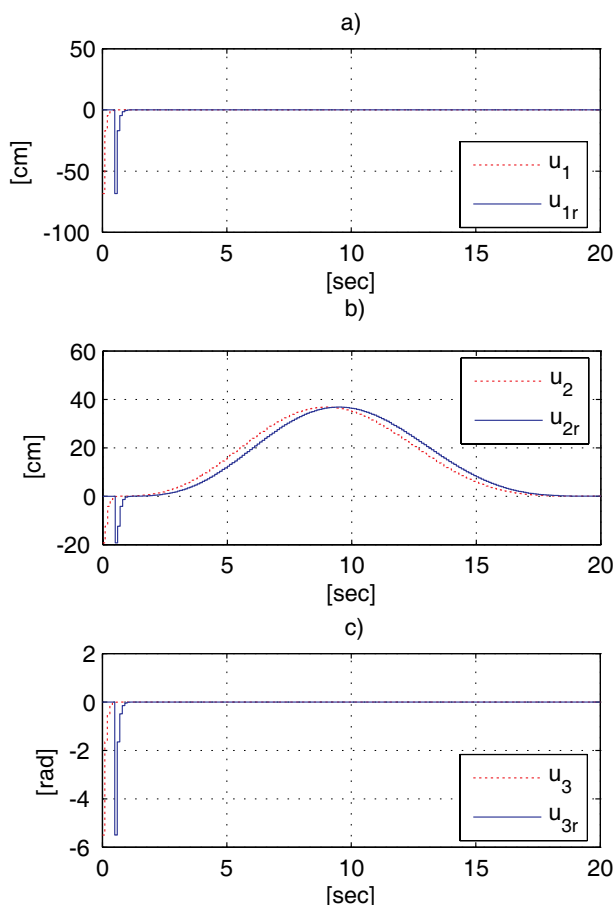


Fig. 8. Control actions u_i and its delayed values $u_{ir} = u_i(t - \tau)$.

To design a discrete-time control strategy is obtained previously the exact discrete-time model of the vehicle which allows in a natural manner the treatments of an input delay at the system. The path-tracking problem is solved by considering a feedback linearization approach that at a first stage provides a noncausal solution. To overcome this causality problem a Smith predictor compensator is implemented in order to approximately estimate the future values of the state needed for the control law. The performance of the propose methodology is evaluated by simulation showing an acceptable error convergence.

REFERENCES

[1] M. Corradini, T. Leo and G. Orlando. Experimental testing of a discrete-time sliding mode controller for trajectory tracking of a

wheeled mobile robot in the presence of skidding effects. *J. of Rob. Sys.*, Vol. 19, pp. 177–188, 2002.

[2] R. Orosco-Guerrero, M. Velasco-Villa and E. Aranda-Bricaire. Discrete-time controller for a wheeled mobile robot. Proc. XI Latin-American Congress of Automatic Control, La Habana, Cuba, 2004.

[3] E. Fabrizi, G. Oriolo, S. Panzieri and G. Ulivi. A kf-based localization algorithm for nonholonomic mobile robots. 6th IEEE Mediterranean Conference on Control and Automation, pp. 130–135, 1998.

[4] M. Wargui, M. Tadjine and A. Rachid. On the stability of an autonomous mobile robot subject to network induce delay. IEEE 5th. International Conference on Control Applications, Hartford, CT, pp. 1329–1334, 1997.

[5] M. Wargui, M. Tadjine and A. Rachid. Stability of real time control of an autonomous mobile robot. IEEE 5th. International Workshop on Robot and Human Communication. Tasukuba, Japan, pp. 311–316, 1999.

[6] V. Kolmanovskii and A. Myshkis, *Applied theory of funcional differential equations*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1992.

[7] J. K. Hale, *Theory of functional differential equations*, Springer-Verlag, New York, 1997.

[8] P. A. Niño-Suárez, M. Velasco-Villa and E. Aranda-Bricaire, Discrete-time feedback linearization of a wheeled mobile robot subject to transport delay. Congreso Latinoamericano de Control Automático, La Habana Cuba, 2006.

[9] P. A. Niño-Suarez, E. Aranda-Bricaire and M. Velasco-Villa, Discrete-time sliding mode path-tracking control for a wheeled mobile robot, 45th IEEE Conference on Decision and Control, pp. 3052-3057, San Diego, CA, USA, 2006.

[10] O. J. M. Smith, Close Control of Loops with Dead Time. *Chem. Eng. Prog.*, Vol. 53, pp. 217-219, 1957.

[11] C. Kravaris and R. A. Wright, Deadtime compensation for nonlinear processes, *AIChE Journal*, Vol. 35, No. 9, pp. 1535–1542, 1989.

[12] M. A. Henson and D. E. Seborg, Time delay compensation for nonlinear processes. *Industrial Engineering Chem.*, Vol. 33, pp. 1493–1500, 1994.

[13] C. Canudas De Wit, B. Siciliano, G. Bastin, B. Brogliato, G. Campion, B. D’Andrea-Novet, A. De Luca, W. Khalil, R. Lozano, R. Ortega, C. Samson and P. Tomei. *Theory of Robot Control*. Springer-Verlag, London, 1996.

[14] G. Campion, G. Bastin and B. D’Andrèa-Novet, Structural properties and classification of kinematic and dynamic models of wheeled mobile robots, *IEEE Transactions on Robotics and Automation*, Vol. 12, No. 1, pp. 47–62, 1996.

[15] R. L. Williams, B. E. Carter, P. Gallina and G. Rosati, Dynamic Model With Slip for Wheeled Omnidirectional Robots, *IEEE Tran. on Robotics and Automation*, Vol 18, No. 3, pp.285–293, 2002.

[16] U. Kotta, *Inversion method in the discrete-time nonlinear control systems synthesis problems*. Springer Berlin-Heidelberg, Berlin, Germany, 1995.

[17] C. H. Moog, R. Castro, M. Velasco and A. Marquez, The disturbance decoupling problem for time-delay nonlinear systems, *IEEE Trans. Aut. Cont.*, vol. 45, pp. 305–309, 2000.

[18] A. Isidori, *Nonlinear control systems*. Springer-Verlag, third edition, 1995.